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AN INTELLIGENT DECISION SUPPORT SYSTEM FOR SOLVING OPTIMIZED GEOMETRIC DESIGN PROBLEMS

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Annotation. An integrated intelligent approach for solving geometric design problems is studied. A general optimization placement problem of arbitrary shaped objects in a bounded container is formulated as a mathematical programming problem in terms of the phi-function technique. Various technological requirements (geometric and mechanical) are considered, including continuous translations and rotations of the objects, allowable distances between objects, prohibited zones in the container, balancing conditions, mechanical strength constraints. Solution strategies, methods, and algorithms to solve different variants of the model are discussed and illustrated with examples.

Keywords: intelligent technology, geometric design, phi-functions, mathematical modeling, nonlinear optimization.

Introduction

One of the tools for studying and optimizing complex technical systems to achieve the state of their optimal functioning is the geometric design technique aimed to modeling and solving optimization placement problems.

These problems are referred to the creation of energy and resource-saving technologies in a wide variety of industries (energy, machinery, ship and aircraft manufacturing, construction, chemical industry, as well as in scientific research in the field of nanotechnology, in modern problems of biology, mineralogy, medicine, materials science in robotics, information encoding problems, image recognition systems, spacecraft management systems) when automating and modeling the processes of placement of 3D solids.

Particularly, the investigation of chromosome organization in biology and medicine is an area of special interest [1]. The chromosome territory can be modeled as an ellipsoid. Chromosomes grow to ellipsoids of different size. The enclosing ellipsoid can be associated with cell nuclei of different size and shape depending on cell types. Chromosome territories can overlap allowing for interaction and co-regulation of different genes. The DNA-free interchromatin channels allow access by regulatory factors to chromosomes movement into a cell nucleus. Smaller nuclei form denser packings, so that fewer channels are available, and the chromosomes near to the center are not accessible to regulatory factors.

It is known that the geometric design problem (known as cutting, packing, layout, loading, covering) are NP-hard. A lot of publications address this class of problems (see, e.g., [2]-[8] and reference therein). Thus, to find its approximate solution, a variety of methods are used: – the simulated annealing algorithm [2] ; – genetic algorithms [3]; – heuristics based on different approximation approaches [4]; – the artificial bee colony algorithm [5]; – monkey algorithm [6], – traditional optimization methods [7], – mixed approaches that apply heuristics and non-linear mathematical programming methods [8].

Mathematical models and solutions methods for the following classes of geometric design problems are developed: – placement of spherical objects [9]; – placement polyhedron objects [10]; – placement objects bounded by spherical, conical and cylindrical surfaces [11]; – balance layout problems [12]; – sparse layout problems [13].

The complexity of an analytical description of the solution space, as well as, the multidimensionality and multiextremality of optimization problems, require the development of modern intelligent technologies to solve them effectively.

The modern method of achieving progress in solving challenging scientific problems is "artificial intelligence". Intellectual information technologies [14] allow us creating intelligent systems that are able to solve creative problems arising in a specific applications.

To develop intelligent technologies, the identification of general patterns of modeling optimization problems of geometric design associated with ensuring the possibility of using different methods and their combinations for solving different classes of the problems.

A unique approach for mathematical and computer modeling proposed in the paper is a power tool for creating an information system for automatic decision-making support in solving the geometric design problems.

Problem formulation

In despite of their different formulations, all the geometric design problems can be described by a general statement, which can be formulated in the following form.

Optimization Placement Problem. Place a given set of objects O_i, $i \in I_n$, into a container taking into account the technological conditions so that the optimization criterion reaches an optimal value.

The input data for the geometric design problem are following:

1) information about a set of objects:
    - spatial shapes: 2D, 3D, nD;
    - simple, composed;
    - regular, irregular;
    - congruent, different;
- ✓ metric characteristics
    - fixed, variable;
    - upper and low bounds;
- ✓ orientation of objects
    - orthogonal orientation;
    - continues rotation;
    - discrete rotation;
    - fixed orientation;
- ✓ transformation type
    - homothetic;
    - stretching;

2) information about placement domain:
- ✓ spatial shapes;
    - 2D, 3D, nD;
    - simple-connected, multi-connected;
- ✓ prohibited zones;
- ✓ placement objects by levels;

metric characteristics
    - fixed, variable;
    - upper and low bounds;
- ✓ transformation type
    - homothetic;
    - stretching (squeezing).

3) technological restrictions on placement which define type of problem:
- ✓ geometric restrictions (for all types of problems)
    - containment of objects to placement domain (packing and layout problems);
    - objects non-overlapping conditions (packing and layout problems);
    - container covering conditions (covering problems); maximum admissible distances between objects (sparse layout problems);
    - minimum admissible distances between objects (layout problems);
    - upper and low bounds of rotations angles of objects (additive manufacturing problems);
- ✓ mechanical restrictions
    - restrictions on behavior system (balance layout problems);
    - mechanical strength constraints (topology optimization of the object shape);
    - influence of physical fields sources (thermodynamic problems);

4) type of optimization criterion:
- ✓ maximization of placed objects;
- ✓ maximization of objects metrical characteristics;
- ✓ maximization of distances between objects;
- ✓ minimization of container metric characteristics;
- ✓ multicriteria optimization.

Depending on the type of the objective function, the problems of optimal placement objects can be either a knapsack problem (KP) with output maximization or an open dimension problem (ODP) with input minimization. In KP, assortment of small items (objects) have to be arranged to a given set of large objects. The value of the packed items should be maximized. In ODP, a set of small items have to be accommodated completely by several large objects for which at least one dimension can be considered as a variable.

We use a special class of point sets, phi-objects, as a mathematical model of real objects. By definition [15] phi-object is non-empty object $O \subset R^d, d = 2{,}3$ which satisfies the following conditions: 1) O is a canonically closed point set; 2) the homotopic type of $int(O)$ coincides with the homotopic type of $cl(O)$). Here $int(O)$ is the interior and $cl(O)$ is of the closure of O.

For example, the set of 3D-objects considered in the paper can be divided into two groups. Convex objects bounded by cylindrical, conical and spherical surfaces belong to the first group. Arbitrary shaped objects approximated by polyhedra form the second group.

Objects can be subjected to congruent, homothetic and stretching transformations. The vector of variables $u_o = (v_o, \theta_o, \lambda_o) \in R^d\ d = 2{,}3$ *is* accociated with the object O, where v_o is a translation vector, θ_o is a vector of rotation angles, and λ_o is a homothety coefficient. The object O translated by the vector v_o, rotated by angles θ_o, subject to homothetic transformation by parameter λ_o, by $O(u) = \{p : p = v_o + \lambda_o \cdot M(\theta_o) \cdot \tilde{p},\ \forall \tilde{p} \in O\}$, where $\tilde{p}$ is an arbitrary point of the object O in its egine coordinate system.

The placement of objects can be constrained by: object orientation (oriented and non-oriented); allowable distances; prohibited zones, balancing conditions.

The set of objects are placed into the container Ω having diferent shapes, such as a cuboid, a sphere, a straight prism with prohibited cylindrical zones and straight rectangular prisms.

The optimization placement problem can be aimed to: minimizing the container or objects metrical characteristics, minimizing the container volume, maximizing the minimal distance between objects, maximizing the number of objects placed into a given container.

1. Phi-function technique.

For analytical description of non-overlapping and containment constraints we use the phi-function technique. Phi-functions allow presenting a mathematical model of the optimization placement problem in the form of mathematical programming problem. The phi-function technique allows us to cover a wide spectrum of packing problems. Paper [15] presents the concept of phi-functions and quasi-phi-functions as an efficient tool for mathematical modeling of placement problems.

Let A and B be two phi-objects. The position of the object A is defined by a vector of placement parameters $u_A = (v_A, \theta_A)$, where: $v_A = (x_A, y_A)$ is a translation vector and θ_A is a vector of rotation parameters.

Phi-functions allow distinguishing the following three cases: $A(u_A)$ and $B(u_B)$ do not intersect, i. e. $A(u_A)$ and $B(u_B)$ do not have common points; $A(u_A)$ and $B(u_B)$ are in contact, i. e. $A(u_A)$ and $B(u_B)$ have only common frontier points; $A(u_A)$ and $B(u_B)$ are intersecting so that $A(u_A)$ and $B(u_B)$ have common interior points.

Definition [15]. A continuous and everywhere defined function $\Phi^{AB}(u_A, u_B)$ is called a phi-function for objects $A(u_A)$ and $B(u_B)$ if

$$\Phi^{AB}(u_A, u_B) < 0, \text{ if } \operatorname{int} A(u_A) \cap \operatorname{int} B(u_B) \neq \varnothing;$$

$$\Phi^{AB}(u_A, u_B) = 0, \text{ if } \operatorname{int} A(u_A) \cap \operatorname{int} B(u_B) = \varnothing \text{ and } frA(u_A) \cap frB(u_B) \neq \varnothing;$$

$$\Phi^{AB}(u_A, u_B) > 0, \text{ if } A(u_A) \cap B(u_B) = \varnothing.$$

It should be noted that inequality $\Phi^{AB}(u_A, u_B) \geq 0$ provides the non-overlapping constraint, i.e., $\operatorname{int} A(u_A) \cap \operatorname{int} B(u_B) = \varnothing$, and inequality $\Phi^{AB^*}(u_A, u_B) \geq 0$ provides the containment constraint $A(u_A) \subset B(u_B)$, i.e. $\mathrm{int}\, A(u_A) \cap \mathrm{int}\, B^*(u_B) = \varnothing$, where $B^* = R^d \setminus \mathrm{int}\, B$.

Definition [21]. A continuous and everywhere defined function $\Phi'^{AB}(u_A, u_B, u')$ is called a quasi phi-function for two objects $A(u_A)$ and $B(u_B)$ if $\max_{u'} \Phi'^{AB}(u_A, u_B, u')$ is a phi-function for the objects.

Here u' is a vector of auxiliary continuous variables that belong to the Euclidean space.

Based on the properties of a quasi-phi-function the non-overlapping constraint can be described in the form: if $\Phi'^{AB}(u_A, u_B, u') \geq 0$ for some u', then $\mathrm{int}\, A(u_A) \cap \mathrm{int}\, B(u_B) = \varnothing$.

2. General Mathematical Model of the Geometric Design Problems.

Let us introduce the variables of the optimization placement problem:

– u_Ω is a vector of metric characteristics of the container Ω;

– $u = (u_1, u_2, \dots, u_n)$ is a vector of placement parameters for objects $O_i, i \in I_n$;

– $u_i = (v_i, \theta_i, g_i)$ is a vector of the placement parameters for the object O_i;

– v_i is the translation vector of the object O_i;

– θ_i is the vector of rotation parameters of the object O_i;

– λ_i is the homothetic coefficient of the object O_i;

– $u' = (u'_{pq}, p \in I_n, q \in I_n, p > q)$ is the vector of additional variables used in quasi-phi-functions for pair of the objects $O_i(u_i)$ and $O_j(u_j)$.

In terms of the phi-function technique, the mathematical model of the general placement problem can be presented in the following form:

$$F(X^*) = \underset{X \in W}{extr}\, F(X), \tag{1}$$

subject to

$$W = \{X \in \mathbb{R}^\sigma : \Psi_s(X) \geq 0, s = 1, \dots, 5\}. \tag{2}$$

In the model (1)-(2):

– $F(X)$ is at least twice-differentiated function; n is the number of objects;

– $n_{qp} = \frac{0.5(1-n_q)}{n_q}$, n_q is the number of objects for which the model uses quasi-phi-functions; n_Ω is the number of variable metric characteristics of the container Ω;

– $X = (u_\Omega, u, u_p) \in \mathbb{R}^\sigma$ is the vector of variables of the problem;

– $\Psi_1(X) \geq 0$ describes the containment condition $O_i \subset \Omega$ for all objects

– $\Psi_1(X) = min\{\Phi^{O_i\Omega^*}(X), i \in I_n\}$;

– $\Phi^{O_i\Omega^*}(X)$ is the phi-function (quasi-phi-function) of the object O_i and $\Omega^* = cl(R^d \setminus \Omega)$;

– $\Psi_2(X) \geq 0$ describes distance (non-overlapping) constraints for the objects $O_i(u_i)$ and $O_j(u_j)$ $\Psi_2(X) = min\{\Phi^{O_iO_j}(X), (i, j) \in I_n\}$;

– $\Phi^{O_iO_j}(X)$ is the phi-function (or quasi-phi-function) for the objects $O_i(u_i)$ and $O_j(u_j)$;

– $\Psi_3(X) \geq 0$ describes distance (non-overlapping) constraints for objects and prohibition zones;

$$\Psi_3(X) = min\{\Phi^{O_iT_k}(X), i \in I_n, k \in I_z\},$$

– $\Phi^{O_iOT_k}(X)$ is the Φ-function for the object $O_i(u_i)$ and the prohibition zone T_k;

– $\Psi_4(X) \geq 0$ describes additional constraints (restrictions on the metric characteristics of the container or placement objects);

– $\Psi_5(X) \geq 0$ describes mechanical conditions.

Different variants of the mathematical model (1)-(2) can be generated depending on the type of the objective function and system of constraints. It makes possible to cover a wide spectrum of geometric design problems.

Let us consider some main features of the mathematical model (1)-(2) that affect the development of a general solution strategy.

1. Model (1)-(2) is formulated in the form of a mathematical programming problem, and involves all its global solutions.

2. The feaseble region W of the problem (1)-(2), in the general case, is a

disconnected set with multi-connected components.

3. The function $\Psi_2(X)$ can be specified by either phi-functions or quasi-phi-functions. The inequality $\Psi_2(X) \geq 0$ can be represented by a) collection of inequality systems with continuously differentiable functions using phi-functions and b) an inequality system using quasi-phi-functions.

4. The feasible region W is described by a system of inequalities of nonsmooth functions that include operators “max” and “min”. Therefore $W = \cup_{l=1}^{m} W_l$, where W_l can be described by a system of inequalities with continuously differentiable functions. Thus, problem (1) - (2) can be represented as

$$F(X^*) = extr\{F(X^{*l}), l = 1,2, \dots, m\},$$

where $F(X^{*l}) = \underset{X \in W_l}{extr} F(X).$

5. If the feasible region of the problem (1) - (2) uses only quasi-phi-functions, it is described by a system of inequalities with continuously differentiable functions.

6. Problem (1) - (2) is NP-hard.

Based on the features (1-7) of the mathematical model (1)-(2) we propose a methodology for solving placement problems, using nonlinear optimization.

1. A Scheme of Intelligent System.

To create an intelligent system a special methodology was developed, which based on the analysis of the input data on the geometric problem can independently make decision on the choice of the necessary mathematical model, the optimal solution strategy, and adjust the parameters of the solution methods depending on the initial data and the specifics of the problem being solved.

It can be seen that the input data for the system are information about the objects being placed, the placement container, technological constraints, and the objective function. As a result, the system outputs the result of solving the problem of geometric design.

The proposed methodology is based on the analysis of the input information about the problem to be solved. It uses several approaches whose fundamental difference is the ability to change the orientation of objects in searching for a solution of the problem, since the arbitrary rotations of the objects make this process much more complex, and require other methods. Because of this, the methodology uses two main approaches to solving problems: that of packing convex objects and that of packing non-oriented objects.

The ability to arbitrarily change the orientation of objects requires the use of other methods of finding initial placements, and, therefore, use of another approach. In the case of non-oriented objects, the approach to solving the problem uses two different strategies to search for an approximation to a global solution, with the strategies being selected depending on the shape of objects. If the objects are convex, a strategy based on homothetical transformations and search for promising points is applied. Since the complexity of the problem increases dramatically in packing non-convex objects, to solve the problem, a multi-stage multi-start strategy is used, which initially uses the strategy of packing non-oriented convex objects.

Each of the proposed strategies is based on the following sequence of stages:

1) construction of starting points belonging to the feasible region;
   - ✓ optimization method for groups of variables;
   - ✓ homothetic transformation;
   - ✓ regular placement method (based on grids);
   - ✓ clusters placement.
2) local optimization;
   - ✓ feasible direction method with e active restriction strategy;
   - ✓ interior point method with special decomposition strategy;
3) global optimization.
   - ✓ successive statistical optimization;
   - ✓ promising point method.

For each of the strategies proposed, there has been developed a set of methods, which takes into account the peculiarities of the problems to be solved.

Let us consider each of these strategies in more detail.

A strategy based on successive statistical optimization.

The main idea of this strategy is to optimize the objective function defined on a set of permutations. To construct admissible starting points from the feasible region, we use methods that use the sequence of objects being placed (optimization method for groups of variables) or the sequence of coordinates of their centers (regular placement method that is focused on the placement of congruent objects). To find local extrema, a modification of method of possible directions was used along with the concept of active inequalities on subregions.

One way to solve multi-extreme problems is the enumeration of local extrema. However, even for a relatively small number of objects, it is impossible to directly enumerate local extrema. Due to the fact that there is a possibility for the above problems to establish a correspondence between the permutations of objects and local extrema, a strategy is applied to find an approximation to a global extremum, which uses the modification method of narrowing neighborhoods. This method is a direct random search which focuses on optimizing the functions that are given on a set of permutations.

The method of narrowing neighborhoods is based on the properties of the probabilistic distribution of local extrema of the objective function. It allows us to search for sequences of the objects to be placed, and obtain an approximation to a global extremum solution of the problem in a relatively short time. To implement it, we need to introduce a certain metric on the permutation set. The search for the best values of the objective function is performed in the neighborhoods given on a set of permutations. At each step of the method, the centers and radii of new neighborhoods are selected based on the statistical information accumulated in the process. If the value of the objective function does not improve during the next search step, then the radii of the neighborhoods decrease.

A strategy based on homothetical transformations and the construction of promising points.

The methods of this strategy use problem dimension increase by introducing variable metric characteristics of objects and homothetic transformations of objects. The strategy is based on the following sequence of steps: 1) a homothetic transformation method to construct initial points; 2) an interior point method together with a decomposition strategy to find local extrema; 3) a method of constructing perspective placements to find approximations to the global extremum.

Since mathematical model (1) - (2) is constructed as a classical nonlinear programming problem, various modifications of nonlinear optimization methods can be applied to solve it. However, to apply numerical nonlinear optimization methods, we must have an admissible starting point. Among the methods used to construct starting points, generally used in object placement problems are various modifications of greedy algorithms. However, since the problems of packing objects are NP-complex, the use of greedy algorithms significantly limits the possibility of enumerating a huge number of local extrema (whose number exceeds $n!$). In addition, the computational cost of constructing starting points increases significantly if objects are allowed continuous rotations.

Using the phi-function method to construct mathematical model (1) - (2) allows us to apply modern methods of nonlinear optimization at all stages of solving the problem. In this regard, a special approach is proposed to construct admissible starting points, with its main idea being to increase the dimension of the problem by introducing variable metric characteristics of objects and homothetic transformations of objects. Suppose that objects are allowed homothetic transformations. For this purpose, we accept the homothetic coefficients to be variable. Then, in order to derive the starting point, a random generation of the placement parameters of the objects to be placed in the container is performed. After that, the problem of nonlinear programming is solved, whose purpose is maximization of sum of homothetic coefficients of all objects. If the solution to this problem results in finding the local maximum point in which all the homothetic coefficients are equal to one, then such a point is taken as the starting point for finding the local extremum of the main problem. It should be noted that, in contrast to the greedy algorithms which are applied for constructing starting points, which can yield good, but identical points, the developed method of constructing starting points allows us to obtain various starting points due to the random method of generating the placement parameters of the of objects.

Since the region of feasible region is determined by a very large number of inequalities, the immediate application of nonlinear optimization methods to find a local extremum will result in considerable computational costs. Therefore, to find the local extrema of formulated optimization problems, a special decomposition method was developed, which allows us to reduce computational costs by significantly reducing the number of inequalities in the process of finding local extrema. Since the region of feasible region is represented as a union of subregions, then it enables to significantly reduce the time for finding the local minimum by reducing it to solving a sequence of subproblems where the feasible subregion is determined by a much smaller number of inequalities. The key idea of the method is to select, at each step, a subregion of the feasible region solutions and to generate subsets of the subregion chosen at each step in this way. Based on the starting point analysis, a system of additional constraints on the placement parameters of each object is added into the problem constraint system, which allows the object is moved within the individual container. Then the inequalities for all the pairs of the objects whose individual containers do not intersect are removed. Thus, we reduce both the number of constraints and, in the case of quasi-phi-functions, and the number of additional variables . Then, a search for the local minimum point for the constructed subproblem executed. The resulting local subproblem extremum is used as the initial point for the next iteration.

Global optimization for this strategy is based on the idea of enumerating local minimums by constructing new promising starting points with using homothetical transformations of objects at the local minimum point obtained. To do this, at the local minimum point, a nonlinear programming problem is solved. As a result of solving such problem, we get the point where we can identify 2 groups of objects: 1) objects that are surrounded by empty space, and therefore, larger objects can be placed instead of these objects; 2) objects which makes it impossible to change over these objects with larger ones to reduce the volume of the container. To determine such appropriate groups of objects, a special auxiliary nonlinear optimization problem is solved, which aims to reduce the volume of the container, provided that the objects placed therein allow homothetical transformations. A peculiarity of the auxiliary problem is the absence of constraints on the maximum value of the homothetic coefficients of objects. As a result, the volume of the container is reduced due to the fact that some objects will be reduced and some, enlarged in sizes. This change in object sizes allows us to define the two groups of objects described above. Since the container volume becomes smaller, some objects become smaller than it is specified, with the next step being to solve an auxiliary problem that will increase the sizes of objects to their specified values. To solve this auxiliary problem, iterative attempts are made to construct a series of starting points, which we will be called promising. To construct such points, we try, in the sequence given, to change over the objects from the first group with the objects from the second group. Such a permutation allows us to go the subregion that is located in the area of gravity of another local minimum. When changing over objects, we reduce their sizes so that they do not overlap with adjacent objects. If it is possible to enlarge the objects to their original sizes, then the point corresponding to this permutation of objects is taken as the starting point for finding a new local minimum of the main problem.

If we cannot find the global extremum of the auxiliary problem of maximizing the sum of the homothetic coefficients of the packed objects from the series of constructed promising points, then the lattest found local minimum is taken as the approximate to a global minimum of the main problem.

The effectiveness of the proposed strategy is achieved through the implementation of successive changes in the dimension of the solution space during the transition between auxiliary problems. The objective function gradually improves due to the fact that the local extremum point of one auxiliary problem is not the local extremum point of another auxiliary problem.

A multi-stage multi-start strategy.

This strategy was used to solve the problem of packing non-convex non-oriented objects. The strategy is focused on finding the optimal placement of non-convex non-oriented objects, which significantly complicates the solution process. Therefore, to reduce large computational costs, we decompose the solution of the problem into several major stages (preparatory and multi-start) and their substages.

Since the strategy is focused on the placement of non-convex non-oriented objects, a clustering method is proposed to construct admissible starting points. Local optimization was performed using an interior point method together with a decomposition strategy. To find local extrema, a multi-start generation of starting points was used.

During the preparatory phase, a number of nonlinear programming problems are solved that allow data to be obtained to construct the starting points of the main packing problem.

At the multi-start stage, both different starting points and corresponding local minimums are constructed. It should be noted that, depending on the shape of clusters, used are either the optimal packing strategy, or strategy of packing parallelepipeds which are allowed orthogonal rotations, or spheres, or non-oriented parallelepipeds and spheres. To solve these problems, we use the strategy based on homothetical transformations and the construction of promising points.

Due to the clustering method of non-convex non-oriented objects, the construction of starting points is reduced to solving the problem of packing twice as little convex objects of a much simpler spatial shape (parallelepipeds and spheres). This greatly reduces the time which needs to construct starting points.

It should be noted that the reduction of computational costs is also facilitated by the fact that the process of finding a local extremum in the problem is divided into two stages: that of solving the linear problem by fixing the angles of rotation and that of solving the nonlinear problem. In addition, since the strategy of finding an approximation to a global extremum is used to accommodate the formed cluster, the constructed starting point is some approximation to the local extremum of the main problem.

As the approximation to a global minimum of the problem is selected as the best local minimum obtained as a result of performing the multi-start phase.

Methods for constructing admissible starting points. To apply local optimization methods, we need to construct starting points that belong to the feasible region of admissible solutions. One of the requirements for the starting point construction methods for object packing problems is to generate a variety of points (which will ensure finding different local extrema) and to reduce computational costs.

Developed in this work are the following methods: the regular placement method for packing congruent objects; the homothetic transformation method for convex objects whose surfaces are formed by conical, cylindrical, and spherical surfaces; and the clustering method for convex polyhedral objects.

Local optimization methods. The analysis of the peculiarities of the mathematical models of packing problems revealed that the feasible region is described by a large number of nonlinear inequalities. This fact requires the development of methods that will effectively succeed the task of large dimensional problems. The main idea of proposed local optimization methods is based on the decomposition of the main problem into subproblems with significantly less the number of both constraints and dimensions. To do this, the following stages are performed: feasible subregions of the feasible region with starting points are sequentially generated; the subsystem of $\varepsilon-$active constraints is singled out; local extrema in the selected subregions are searched for with the help of modern second-order NLP solvers; transition to other subregions is organized.

Conclusions

The paper proposes intellectual technologies based on a unique approach to mathematical and computer modeling of the class of placement problems.

A methodology for solving geometric design problems is proposed.

The methodology focuses on advanced developments in geometric design and the use of powerful software packages to solve mathematical programming problems.

The effectiveness of the proposed tools is confirmed by a number of computational results that compared with those found by the other researchers.

Optimized packing models considered in this work are referred to large-scale nonlinear programming problems. Aggregation/decomposition techniques are aimed to use a special structure of the problem and may provide a reasonable alternative to direct solution.

References

1. Wang, Y., Nagarajan, M., Uhler, C., Shivashankar, G. V. (2017) Orientation and repositioning of chromosomes correlate with cell geometry–dependent gene expression. *Molecular Biology of the Cell*, 28(14), 1997–2009. https://doi.org/10.1091%2Fmbc.E16-12-0825.

2. Cao, P., Fan, Z., Gao, R., Tang, J. (2016) Complex Housing: Modeling and Optimization Using an Improved Multi-Objective Simulated Annealing Algorithm. *Proc. of ASME*, 60563, V02BT03A034.

3. Egeblad, J, Nielsen, B.K., Odgaard, A. (2007) Fast neighborhood search for two- and three-dimensional nesting problems. *European Journal of Operational Research*, 183(3), 1249–1266.

4. Litvinchev, I., Ozuna, L. (2014) Approximate packing circles in a rectangular container: valid inequalities and nesting. *Journal of Applied Research and Technology*, 12(4), 716–723.

5. Guangqiang, L. A., Fengqiang, Z., Rubo, Z., Du, Jialu Du., Chen, G., Yiran, Z. (2016) Parallel Particle Bee Colony Algorithm Approach to Layout Optimization. Journal of Computational and Theoretical Nanoscience, 13(7), 4151–4157.

6. Torres-Escobar, R., Marmolejo-Saucedo, J.A., Litvinchev, I. (2020) Binary monkey algorithm for approximate packing non-congruent circles in a rectangular container. Wireless Networks, 26(7): 4743–4752. https://doi.org/10.1007/s11276-018-1869-y.

7. Birgin, E.G., Lobato, R.D., Martinez, J.M. (2016) Packing ellipsoids by nonlinear optimization. Journal of Global Optimization, 65, 709–743.

8. Fasano, G. (2013) A global optimization point of view to handle non-standard object packing problems. Journal of Global Optimization, 55(2), 279 – 299.

9. Stoyan, Y., Yaskov, G., Romanova, T., Yakovlev, S., Cantú, J.M.V. (2020) Optimized packing multidimensional hyperspheres: A unified approach. Mathematical Biosciences and Engineering, 17(6), 6601–6630.

10. Romanova, T., Bennell, J., Stoyan, Y., Pankratov, A. (2018) Packing of concave polyhedra with continuous rotations using nonlinear optimisation. European Journal of Operational Research, 268(1), 37–53.

11. Stoyan, Y.G., Semkin, V.V., Chugay, A.M. (2016) Modeling Close Packing of Objects. Cybernetics and Systems Analysis, 52(2), 296–304.

12. Stoyan, Y., Romanova, T., Pankratov, A., Kovalenko, A., Stetsyuk, P. (2016) Balance layout problems: Mathematical modeling and nonlinear optimization. Springer Optimization and Its Applications, 114, 369–400.

13. Romanova, T., Pankratov, A., Litvinchev, I., Plankovskyy, S., Tsegelnyk, Y., Shypul O. (2021) Sparsest packing of two-dimensional objects, International Journal of Production Research, 59(13), 3900–3915.

14. Pashchenko, F. F., An, B. T., Hieu, T. D., Pashchenko, A. F. and Van Trong, N. (2020) Intelligent Technologies in Decision-Making Support Systems. Proc. of the International Conference Engineering and Telecommunication (En&T), 1-4, doi: 10.1109/ EnT50437.2020.9431248.

15. Stoyan, Y., Romanova, T. (2012) Mathematical Models of Placement Optimisation: Two- and Three-Dimensional. In: G. Fasano and J. Pintér (eds) Modeling.

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